COMPARISON OF SHRUNKEN ESTIMATORS OF THE SCALE PARAMETER OF AN EXPONENTIAL DENSITY FUNCTION TOWARDS AN INTERVAL

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SUMMARY

A variety of shrunken estimators have been considered for the estimation of scale parameter of an exponential density function when a prior or guess interval containing the parameter θ is available. Comparisons with the minimum mean squared error esstimator $\frac{n}{(n+1)}\bar{x}$, in terms of mean squared error have been made. It is shown that these estimators are preferable than $\frac{n}{(n+1)}x$ in some guessed interval of the parameter.

1. INTRODUCTION

In the estimation of an unknown parameter there often exists some prior knowledge about the parameter which one would like to utilize in order to get a better estimate. The Bayesian approach is well known example in which prior knowledge about the parameter is available in the form of prior distribution.

According to Thompson [1] some times a natural origin θ_0 is there such that one would like to that the minimum variance unbiased linear estimator (MVULE) $\hat{\theta}$ for θ and to move it close to θ_0. This leads to a shrunken estimator for θ which is better than $\hat{\theta}$ near θ_0 and possibly worse than $\hat{\theta}$ farther away from θ_0 (measured in terms of mean squared error). Thompson [2] extended this result and shrunk the minimum variance unbiased estimator of the mean of a normal ditribution towards an interval.

In this paper we have considered the estimation of scale parameter θ in exponential density function when a guess or prior is available in the form of an interval (θ_1, θ_2) which contains θ in it. We have considered four types of estimators and have obtained expressions for the mean squared error of these estimators for some selected values of n, $\frac{\theta_1}{\theta}$, $\frac{\theta_2}{\theta}$ and k. Comparisons with the minimum mean squared error estimator $\frac{n}{(n+1)}\bar{x}$, have been made and it is shown that these estimators have smaller mean squared error than the estimator $\frac{n}{(n+1)}\bar{x}$ in certain range of the parameter space.

1. Different Estimators Towards a point θ_o

1.1 Estimator T_L :

Let $x_1, x_2, \ldots\ldots, x_n$ be a random sample of size n from an exponential density

$$f(x,\theta)=\frac{1}{\theta}e^{-x/\theta}, \qquad x>0,\ \theta>0. \qquad \ldots (1)$$

The maximum likelihood estimate of the scale parameter θ is the sample mean

$$\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i.$$

Suppose a guessed value θ_0 of θ is available. Following Pandey [3] an estimator for θ can be written as

$$T=a\,[k\bar{x}+(1-k)\,\theta_o] \qquad \ldots (2)$$

where $0\leqslant a\leqslant 1$ and k is a constant between zero and one to be specified by the experimenter according to his belief in θ_o. A value of k near zero implies strong belief in θ_o. Now,

$$MSE(T)=a^2k^2\,Var\,(\bar{x})+\theta^2\,(ad-1)^2 \qquad \ldots (3)$$

where $d=k+(1-k)\,p$ with $p=\frac{\theta_o}{\theta}$. $MSE(T)$ is a function of p, a and k jointly. Analytically, the simultaneous values of a and k which will minimize $MSE(T)$ cannot be found. Therefore, for a variety of values of p, a and k, $MSE(T)$ has been calculated and the best choice for a is $a=1$. Thus, the proposed estimator is

$$T_L=k\bar{x}+(1-k)\theta_o. \qquad \ldots (4)$$

Further more, it appeared that for p close to one, k should be as small as possible, but for p far from one, k should be large. If $p=1$, take $a=1$ and $k=0$, thus $T_L=\theta_o$. But this is obvious, because if we know θ we do not need to estimate it. In practice we have to weigh our confidence $(1-k)$ in θ_o against the risk of being far out. So, the value of k should be chosen according to the confidence in the guessed values θ_o. The more confidence in θ_o will imply the smaller values of k.

1.2 Estimator T_T :

Thompson [1] considered the estimator T_L and determined the value of k for which $MSE(T_L)$ is minimum. Such a value of k is $k_{min}=(\theta-\theta_o)^2/((\theta-\theta_o)^2+\theta^2/n)$ which depends upon θ. If we replace θ by its consistent estimator $\bar{x}$ and θ^2 by its consistent estimator $\bar{x}^2$, the estimate of k_{min} is

$$\hat{k}_{min}=\frac{(\bar{x}-\theta_o)^2}{(\bar{x}-\theta_o)^2+\bar{x}^2/n}. \quad \dots (5)$$

Whence the point shrunken estimator towards the point θ_o is

$$T_T=\frac{(\bar{x}-\theta_o)^3}{(\bar{x}-\theta_o)^2+\frac{\bar{x}^2}{n}}+\theta_o \quad \dots (6)$$

1.3 Estimator T_P :

The method proposed by Pandey [3] is to considered k as a constant and to find the value of a for which $MSE(T)$ is minimum. Such a value of a is

$$a_{min}=\frac{[k\theta+(1-k)\,\theta_o]\,\theta}{[k\theta+(1-k)\,\theta_o]^2+\frac{k^2\theta^2}{n}} \quad \dots (7)$$

which depends upon the unknown parameter θ. If we replace θ by its consistents estimator $\bar{x}$ and θ^2 by its consistent estimator $\bar{x}^2$, we get an estimate of a_{min} as

$$\hat{a}_{min}=\frac{[k\bar{x}+(1-k)\,\theta_o]\,\bar{x}}{\{k\bar{x}+(1-k)\theta_o\}^2+\frac{k^2\bar{x}^2}{n}}. \quad \dots (8)$$

Whence the shrunken estimator towards a point θ_o is

$$T_P=\frac{[k\bar{x}+(1-k)\,\theta_o]^2\,\bar{x}}{[k\bar{x}+(1-k)\,\theta_o]^2+\frac{k^2\bar{x}^2}{n}} \quad \dots (9)$$

This estimator includes both the estimators $\bar{x}$ and $\frac{n}{(n+1)}\bar{x}$ as the special case for $k=0$ and $k=1$ respectively. In practice, the value of k is determined by the experimenter according to his belief in the guessed value θ_o either due to his past experience or with the help of experimental materials. For example, suppose a factory is producing electric bulbs whose life times are exponentially distributed with mean life time θ. From past data the mean life time say θ_o is known and the [experimenter is of 90% confident that the mean life time has not changed. Therefore he will take the value of k as .90.

1.4 Estimator T_w :

Following Pandey [3] a shrunken estimator for θ towards a point θ_o is proposed as follows :

$$T_w=(\bar{x})^k\,(\theta_o)^{(1-k)}. \qquad \ldots(10)$$

This estimator also behaves like other estimator proposed in previous sections. If $\frac{\theta_o}{\theta}\simeq 1$, the smaller value of k give better result and if the difference between θ_o and θ is too far, larger values of k are preferable.

2. Shrunken Estimators Towards an Interval (θ_1, θ_2)

Consider the situation where we have an interval (θ_1, θ_2) as a guess of θ rather than a point θ_o The shrunken estimators in this situation can be obtained as follows :

(a) Suppose θ_1 and θ_2 are the equal probable values of θ_o. The simple average of the point shrunken estimators obtained by replacing θ_o in T_T by θ_1 and θ_2 respectively, will give the shrunken estimator towards an interval. Thus the resulting estimator is

$$M_T=\tfrac{1}{2}\left[\frac{(\bar{x}-\theta_1)^3}{(\bar{x}-\theta_1)^2+\frac{\bar{x}^2}{n}}+\frac{(\bar{x}-\theta_2)^3}{(\bar{x}-\theta_2)^2+\frac{\bar{x}_2}{n}}+\theta_1+\theta_2\right] \qquad \ldots(11)$$

(b) Take the mean value of point shrunken estimator T_T with equal weights at equal intervals in (θ_1, θ_2). The resulting estimator is

$$M_{T1}=\bar{x}+\frac{\bar{x}^2}{2n(\theta_2-\theta_1)}\log\left\{\frac{(\bar{x}-\theta_2)^2+\frac{\bar{x}^2}{n}}{(\bar{x}-\theta_1)^2+\frac{\bar{x}^2}{n}}\right\} \qquad \ldots(12)$$

(c) Take the mean value of the point shrunken estimator T_L with equal weights at equal interval in (θ_1, θ_2). The resulting estimator is

$$M_L = k\bar{x} + (1-k)\left(\frac{\theta_1+\theta_2}{2}\right). \qquad \ldots(13)$$

(d) Take the mean value of the point shrunken estimator T_P with equal weights at equal intervals in (θ_1, θ_2). The resulting estimator is

$$M_P = \bar{x} - \frac{k\bar{x}^2}{\sqrt{n}\,(1-k)(\theta_2-\theta_1)}\left\{\text{arc tan}\left(\frac{k\bar{x}+(1-k)\,\theta_2}{k\bar{x}/\sqrt{n}}\right) - \text{arc tan}\left(\frac{k\bar{x}+(1-k)\theta_1}{k\bar{x}/\sqrt{n}}\right)\right\} \qquad \ldots(14)$$

(e) Take the mean value of the point shrunken estimator T_W with equal weights at equal intervals in (θ_1, θ_2). The resulting estimator is

$$M_W = \frac{(\theta_2^{(2-k)} - \theta_1^{(2-k)})\bar{x}^k}{(\theta_2-\theta_1)(2-k)}. \qquad \ldots(15)$$

Now, the estimator M_L is identical to T_L if we have $\theta_0 = \frac{\theta_1+\theta_2}{2}$. So in M_L only the centre of the interval is of importance, not the end point as such. In M_T the end points are of importance. If $k=o$, $M_W = \frac{\theta_1+\theta_2}{2}$ and if $k=1$, $M_W=\bar{x}$. Therefore, M_L and M_W appear to be identical at these points.

3. Comparisons of Different Proposed Estimators

Since $\frac{2n\bar{x}}{\theta}$ follows a chi-square distribution with $2n$ degrees of freedom, the density of $\bar{x}$ is

$$f(\bar{x}, \theta) = \frac{n^n}{\theta^n \Gamma(n)} e^{\frac{-n\bar{x}}{\theta}} (\bar{x})^{n-1}\, d\bar{x};\ \bar{x} > o,\ \theta > o. \qquad \ldots(16)$$

We have,

$$MSE(M_T) = \frac{\theta^2}{4n}\left[\, n\left(\frac{\theta_1+\theta_2}{\theta} - 2\right)^2 + \frac{1}{n\Gamma(n)}\int_0^\infty\right.$$

$$\left\{\frac{\left(u-n\frac{\theta_1}{\theta}\right)^3}{\left(u-n\frac{\theta_1}{\theta}\right)^2+u^2/n}+\frac{\left(u-n\frac{\theta_2}{\theta}\right)^3}{\left(u-n\frac{\theta_2}{\theta}\right)^2+u^2/n}\right\}^2 e^{-u}u^{n-1}du+$$

$$2\left(\frac{\theta_1+\theta_2}{\theta}-2\right)\frac{1}{\Gamma(n)}\int_0^\infty\left\{\frac{\left(u-n\frac{\theta_1}{\theta}\right)^3}{\left(u-n\frac{\theta_1}{\theta}\right)^2+u^2/n}\right.$$

$$\left.+\frac{\left(u-n\frac{\theta_2}{\theta}\right)^3}{\left(u-n\frac{\theta_2}{\theta}\right)^2+u^2/n}\right\}e^{-u}\,u^{n-1}\,du.\Bigg] \qquad \ldots(17)$$

$$MSE(M_L)=\frac{\theta^2}{n}\left[k^2+n(1-k)^2\left(\frac{\theta_1+\theta_2}{2\theta}-1\right)^2\right] \qquad \ldots(18)$$

$$MSE(M_P)=\frac{\theta^2}{n}\left[1+\frac{1}{n^3\Gamma(n+1)\left(\frac{1}{k}-1\right)^2\left(\frac{\theta_2-\theta_1}{\theta}\right)^2}\right.$$

$$\int_0^\infty\left\{\arctan\left(\frac{u+\left(\frac{1}{k}-1\right)n\frac{\theta_2}{\theta}}{u/\sqrt{n}}\right)-\arctan\left(\frac{u+\left(\frac{1}{k}-1\right)n\frac{\theta_1}{\theta}}{u/\sqrt{n}}\right)\right\}^2$$

$$e^{-u}\,u^{(n+3)}\,du-\frac{2}{n^{3\,2}\,\Gamma(n+1)\left(\frac{1}{k}-1\right)\left(\frac{\theta_2-\theta_1}{\theta}\right)}$$

$$\int_0^\infty\left\{\arctan\left(\frac{u\;.\;(1/k-1)n\frac{\theta}{\theta}}{u/\sqrt{n}}\right)-\arctan\left(\frac{u+\left(\frac{1}{k}-1\right)n\frac{\theta_1}{\theta}}{u/\sqrt{n}}\right)\right\}$$

$$e^{-u}\,u^{(n+2)}\,du+\frac{2}{\sqrt{n}\;\Gamma(n+1)\left(\frac{1}{k}-1\right)\left(\frac{\theta_2-\theta_1}{\theta}\right)}$$

$$\int_0^\infty\left\{\arctan\left(\frac{u+\left(\frac{1}{k}-1\right)n\frac{\theta_2}{\theta}}{u/\sqrt{n}}\right)-\arctan\left(\frac{u+\left(\frac{1}{k}-1\right)n\frac{\theta_1}{\theta}}{u/\sqrt{n}}\right)\right\}$$

$$\left.e^{-u}\,u^{(n+1)}\,du\right]$$

$$MSE(M_W)=\frac{\theta^2}{n}\left[\frac{\left[\left(\frac{\theta_2}{\theta}\right)^{(2-k)}-\left(\frac{\theta_1}{\theta}\right)^{(2-k)}\right]}{(2-k)n^{(k-1)}\left(\frac{\theta_2-\theta_1}{\theta}\right)\Gamma(n)}\right.$$

$$\left.\left\{\frac{\left[\left(\frac{\theta_2}{\theta}\right)^{(2-k)}-\left(\frac{\theta_1}{\theta}\right)^{(2-k)}\right]\Gamma(n+2k)}{(2-k)\,(n^k)\left(\frac{\theta_2-\theta_1}{\theta}\right)}-2\,\Gamma(n+k)\right\}+n\right] \quad ...(20)$$

$$MSE\left(\frac{n}{(n+1)}\bar{x}\right)=\frac{\theta^2}{(n+1)}\Big) \quad ...(21)$$

Properties of the estimator M_{T_1} has been studied in a separate paper, therefore the expression for $MSE\ (M_{T_1})$ has not been given here. The relative efficiency of these estimators with respect to minimum mean squared error estimator $\frac{n}{(n+1)}\bar{x}$ is defined as

$$REF\left(M_i,\ \frac{n}{(n+1)}\bar{x}\right)=\frac{MSE\left(\frac{n}{(n+1)}\bar{x}\right)}{MSE\,[M_i)},\ i=T.L,P \text{ and } W \quad ...(22)$$

The integrals involved in the expressions of mean squared errors can be evaluated by numerical quadrature methods. We have evaluated these by using the 10-points Gauss-Laguerre quadrature formula. The calculations of the relative efficiencies have been done for different values of $n, \frac{\theta_1}{\theta}, \frac{\theta_2}{\theta}$ — and k and are shown in Table 1.

From Table 1, we observe the following:-

(*i*) The relative efficiency of M_L reaches at the maximum when $\theta=\frac{\theta_1+\theta_2}{2}$ and it decreases as the difference between θ and $\frac{\theta_1+\theta_2}{2}$ increases.

(*ii*) If k is small, *i.e.*, we have more confidence in the guess interval $(\theta_1,\ \theta_2)$, M_L is generally the best estimator, followed by M_W and M_T.

(*iii*) If k is moderate *i.e.* $(.50 < k \leqslant .75)$, M_T and M_W are preferable.

(*iv*) If k is near to one *i.e.*, we have not much confidenee in our guessed interval $(\theta_1,\ \theta_2)$, the estimator M_P reduces to $\frac{n}{(n+1)}\bar{x}$ and is preferable.

TABLE 1

The Relative Efficiencies of M_T , M_P , M_L and M_W with respect to $\frac{n}{(n+1)}\ \bar{X}$

n	$\frac{\theta_1}{}$	$\frac{\theta_2}{\theta}$	M_T	$k=.25$			$k=.50$			$k=.75$		
				M_P	M_L	M_W	M_P	M_L	M_W	M_P	M_L	M_W
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
	.20	.30	1.0176	1.0582	0.7413	0.5738	1.0915	1.1163	0.8579	1.0490	1.1228	1.3303
	.30	.45	1.0979	1.0070	1.0393	0.8542	1.0958	1.3813	1.2454	1.0653	1.1798	1.4952
	.50	.75	1.4827	0.9283	2.5016	2.2130	1.0746	2.1099	2.3030	1.0858	1.2737	1.5666
	.75	1.12	1.9005	0.8703	10.8551	9.8078	1.0321	2.9553	3.1382	1.0957	1.3316	1.4357
	.90	1.35	1.7744	0.8480	9.4488	8.7318	1.0070	2.8657	2.7929	1.0958	1.3265	1.3271
	1.00	1.50	1.5735	0.8365	4.4651	4.9631	0.9913	2.5263	2.3994	1.0943	1.3061	1.2553
	.20	.40	1.0447	1.0378	0.8433	0.6683	1.0244	1.2146	0.9952	1.0558	1.1462	1.4061
	.30	.60	1.2232	0.2810	1.3090	1.1010	1.0916	1.5738	1.5172	1.0795	1.2112	1.5437
3	.50	1.00	1.6054	0.9033	4.4651	3.9587	1.0582	2.1099	2.8057	1.0908	1.2737	1.5321
	.75	1.50	1.6379	0.8504	8.4395	8.8499	1.0080	2.6953	2.8128	1.0951	1.3316	1.3332

	.90	1.80	1.3928	0.8308	6.5645	3.2542	0.9810	2.8657	2.1109	1.0916	1.2810	1.2068
	1.00	2.00	1.2099	0.8209	15484	1.8673	0.9648	1.7143	1.7007	1.0877	1.2308	1.1293
	.20	.80	1.3570	0.9706	1.5484	1.2946	1.0853	1.7143	1.6762	1.0750	1.2308	1.5571
	.30	1.20	1.4606	0.9116	4.4651	3.7954	1.0580	2.5263	2.7431	1.0882	1.3061	1.5420
	.50	2.00	1.1116	0.8475	4.4651	5.3889	0.9968	2.5263	2.5158	1.0908	1.3061	1.2865
	.75	3.00	0.9239	0.8102	0.5537	0.7395	0.9378	0.9099	1.0870	1.0729	1.0622	0.9902
	.90	3.60	0.8882	0.7974	0.2779	0.3876	0.9116	0.5275	0.7220	1.5380	0.8768	0.8604
	1.00	4.00	0.8704	0.7911	0.1943	0.2786	0.8979	0.3871	0.5747	1.0475	0.7916	0.7899
	.20	.30	0.9476	1.0211	0.3842	0.2936	1.0473	0.7088	0.4656	1.0274	1.0821	0.9539
	.30	.45	0.9667	0.9929	0.5467	0.4454	1.0466	0.9372	0.7239	1.0358	1.1931	1.2461
	.50	.75	1.2595	0.9531	1.4200	1.2633	1.0310	1.7638	1.7281	1.0453	1.4022	1.6179
	.75	1.12	1.8271	0.9256	11.2552	10.4725	1.0064	3.4068	3.5409	1.0480	1.5508	1.6274
	.90	1.35	1.6896	0.9155	8.5890	7.5384	0.9929	3.1549	3.0550	1.0466	1.5369	1.4994
	1.00	1.50	1.4716	0.9103	2.8354	3.1436	0.9847	2.4348	2.3599	1.0449	1.4835	1.3979
7	.20	.40	0.9551	1.0097	0.4393	0.3441	1.0477	0.7900	0.5526	1.0310	1.1263	1.0697
	.30	.60	1.0575	0.9796	0.6980	0.5836	1.0429	1.1227	0.9323	1.0393	1.2593	1.3856
	.50	1.00	1.5357	0.9413	2.8354	2.5522	1.0213	1.7638	2.5228	1.0469	1.4022	1.6716
	.75	1.50	1.4014	0.9167	7.0551	7.7327	0.9936	3.4068	3.0916	1.0463	1.5509	1.5075

TABLE 1—*Contd.*

1	2	3	4	5	5	7	8	9	10	11	12	13
	.90	1.80	1.2321	0.9080	3.3882	1.8508	0.9796	3.1549	1.9125	1.0429	1.4202	1.3262
	1,00	2.00	1.1107	0.9036	0.8358	0.9842	0.9714	1.2728	1.3729	1.0399	1.3024	1.2095
	.20	.80	1.2568	0.9751	0.8358	0.6951	1.0389	1.2728	1.0663	1.0403	1.3024	1.4408
	.30	1.20	1.3408	0.9461	2.8354	2.4183	1 0218	2.4348	2.4023	1.0456	1.4835	1.6660
	.50	2.00	0.9174	0.9161	2.8354	3,5154	0.9885	2.4348	2.5567	1.0433	1.4835	1.4430
	.75	3.00	0.9863	0.8994	0.2844	0.3693	0.9588	0.5504	0.7433	1.0308	0.9750	1.0025
	.90	3,60	1.0999	0.8939	0.1408	0.1908	0.9461	0.2922	0.4488	1.0218	0.7022	0.8198
	1.00	4.00	1.1293	0.8912	0.0981	0.1366	0.9392	0.2090	0.3437	1.0158	0.5657	0.7266
	.20	.30	0.9484	0.9960	0.1950	0.1482	1.0048	0 3973	0.2412	0.9930	0.8603	0.5819
	.30	.45	0.9358	0.9843	0.2792	0.2265	1.0056	0.5467	0.3877	0.9977	1.0095	0.8618
	.50	.75	1.1245	0.9680	0.7506	0.6686	0.9999	1.2060	1.0867	1.0034	1.3502	1.4512
	.75	1.12	1.7905	0.9568	9.8211	9.2512	0.9899	3.5424	3.6433	1.0056	1.6559	1.7177
	.90	1.35	1.7406	0.9528	6.3830	5.1780	0.9843	3.0380	2.9708	1.0056	1.6244	1.5721

	1.00	1.50	1.4141	0.9508	1.5894	1.7422	0.9810	1.9355	1.9523	1.0051	1.5095	1.4257
	.20	.40	0.9432	0.9913	0.2234	0.1741	1.0055	0.4491	0.2894	0.9950	0.9174	0.6828
	.30	.60	0.9885	0.9789	0.3585	0.2988	1.0044	0.6773	0.5142	0.9998	1.1080	1.0345
	.50	1.00	1.4813	0.9632	1.5894	1.4372	0.9960	1.2060	1.8731	1.0046	1.3502	1.6419
15	.75	1.50	1.2476	0.9533	4.8241	5.3700	0.9847	3.5424	3.0319	1.0054	1.6559	1.5832
	.90	1.80	1.2076	0.9499	1.7163	0.9733	0.9789	3.0380	1.4318	1.0044	1.3841	1.3198
	1.00	2.00	1.1885	0.9482	0.4316	0.5011	0.9755	0.7895	0.9169	1.0033	1.1765	1.1505
	.20	.80	1.2503	0.9772	0.4316	0.3579	1.0028	0.7895	0.6000	1.0044	1.1765	1.1139
	.30	1.20	1.3162	0.9653	1.5894	1.3530	0.9962	1.9355	1.7342	1.0039	1.5095	1.6086
	.50	2.00	0.9028	0.9532	1.5894	1.9795	0.9827	1.9355	2.2130	1.0043	1.5095	1.4918
	.75	3.00	1.0472	0.9466	0.1437	0.1842	0.9705	0,3004	0.4375	0.9997	0.7323	0.8727
	.90	3.60	1.1055	0.9445	0.0708	0.0947	0.9653	0.1535	0.2492	0.9962	0.4625	0.6571
	1.00	4.00	1.0159	0.9435	0.0492	−0.0677	0.9625	0.1079	0.1869	0.9938	0.3509	0.5578

(*v*) The relative efficiencies of different estimators decrease as the sample size is increased implying that the proposed estimators are preferable for smaller sample sizes.

3. Conclusions

We conclude that M_L is a useful estimator if

(*i*) k is small (*i.e.* $0 \leqslant k \leqslant .25$)

(*ii*) $.50 \leqslant \frac{\theta_1+\theta_2}{2\theta} \leqslant 1.25$

and

(*iii*) sample size n is small.

Similarly, the estimator M_T and M_W are useful estimators if (*i*) $.50 \leqslant k \leqslant .75$, (*ii*) $\frac{\theta_1+\theta_2}{2\theta} \leqslant .50$ and $\frac{\theta_1+\theta_2}{2\theta} \geqslant 1.25$ and (*iii*) sample size n is small.

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